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The Advantages of Dynamic Stiffness Parameters Over Classical Bearing Coefficients

An ideal model for a rotor/bearing system will enjoy several properties. First, it will have parameters that are independent of coordinate system orientation. It will be capable of handling isotropic as well as anisotropic systems. It will encompass fluid as well as rotor dynamic characteristics in a single, elegant model. Finally, it will be useful for analyzing and predicting all types of stability problems, particularly fluid-induced instabilities which are a very common problem in rotating machinery.

This article takes such a model, developed by Don Bently and Dr. Agnes Muszynska, and shows how its Dynamic Stiffness parameters provide significant benefits over the bearing coefficients used in classical models. The relationship between Dynamic Stiffness parameters and classical bearing coefficients is shown, as well as techniques for transforming between the two for a general, anisotropic bearing.

"One drawback of such coefficients, however, is that it makes prediction of fluid-induced instability difficult."

The Bently/Muszynska Model

The fluid force model developed by Bently and Muszynska [1] pertains to rotors at low and medium eccentricities. Among other contributions, this model has clarified understanding of self-excited rotor vibrations such as oil whirl and whip. An isotropic bearing is approximated by the Bently/Muszynska model as follows:

$$-\boldsymbol{F}_f = \underbrace{m_f\left(\ddot{\boldsymbol{r}} - j2\lambda\Omega\dot{\boldsymbol{r}} - \lambda^2\Omega^2\boldsymbol{r}\right)}_{=0 \text{ if the fluid inertia effect } m_f = 0 \text{ or is negligible}} + D\dot{\boldsymbol{r}} + \left(K - j\lambda\Omega D\right)\boldsymbol{r} \qquad \textbf{(1)}$$

Here, F_f denotes the fluid force, m_f denotes the fluid inertia effect, K and D are the fluid stiffness and damping, Ω is shaft speed, and λ is the fluid circumferential average velocity ratio.

ELLIPTIC ORBIT AS A COMBINATION OF FORWARD AND REVERSE COMPONENTS

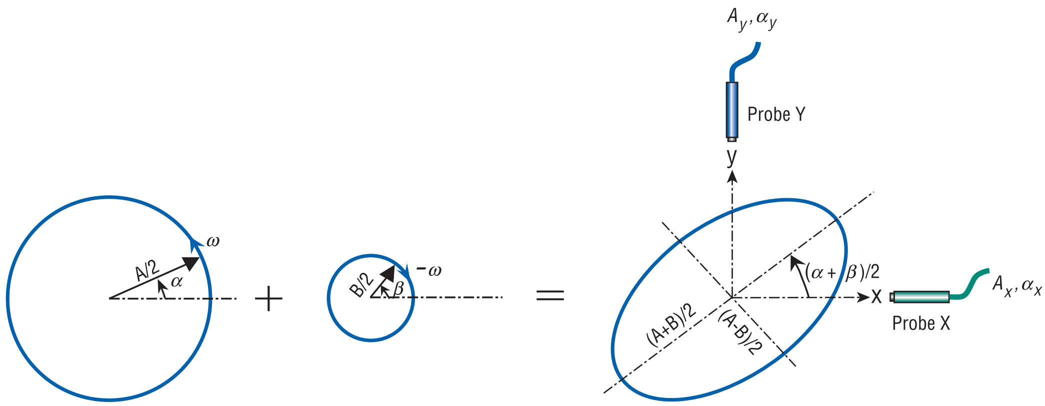

FIGURE 1

The rotor lateral response is described by the vector of displacement $r = x + jy$ (x and y are horizontal and vertical displacements, respectively, and $j = \sqrt{-1}$).

Equation (1) represents a fluid force rotating forward at average speed $\lambda\Omega$ with stiffness K, damping D, and the fluid inertia effect m_f . Unlike classical bearing coefficients, Dynamic Stiffness parameters such as the previously mentioned K, D, λ (and reverse stiffness and damping, K' and D', to be introduced later) are independent of x,y coordinate system orientation.

"Another drawback of bearing coefficients is that they are not independent of the coordinate system orientation."

In general, fluid bearings attached to a rotor are anisotropic. Thus, a model which includes the effects of asymmetry to analyze a general anisotropic bearing is desirable for the most accurate results. A more generalized version of the Bently/Muszynska model of equation (1) does include anisotropic effects, and is shown later in this article. This model, through its use of the parameter λ, clearly indicates the instability margin, even for rotors with anisotropic bearings [2].

Bearing Coefficients

In practice, some engineers use bearing coefficients [3] from manuals or handbooks for rotor dynamic analysis. This is particularly commonplace when measured data is not available. One drawback of such coefficients, however, is that it makes prediction of fluid-induced instability difficult. Some have argued that fluid-induced instability is not nearly the problem that it was fifty years ago. This is not true. While it may show up less frequently in operating machines today, that is primarily because designers consider it more fully in their designs and take measures to avoid it, often by compromising other desirable attributes of the rotor dynamic system and by using less-than-ideal bearing designs in an attempt to preclude these instabilities. Because designers must be concerned with such instabilities, the ability for a model to fully address and predict fluid-induced instabilities remains vital.

Another drawback of bearing coefficients is that they are not independent of the coordinate system orientation. This can complicate data collection and reduction, as excitation and measurement locations do not always coincide with the most desirable coordinate system orientations. The use of a single complex variable, r, instead of bearing coefficients with x and y

variables, is a better and simpler way to model the bearing force and the rotor response.

Transformation between Dynamic Stiffness and Classical Bearing Coefficients

As noted earlier, this article establishes a method of transformation between Dynamic Stiffness parameters and classical bearing coefficients for a general anisotropic bearing. This is developed as a series of steps. First, the elliptic response of the rotor due to anisotropic bearings is described using the concepts of full spectrum, whereby any generalized elliptical response can be expressed as a linear summation of forward- and reverse-rotating vectors of various rotational frequencies. Next, the fluid force in an anisotropic bearing is introduced using Dynamic Stiffness parameters, followed by the response solutions expressed in terms of their forward and reverse (i.e., full spectrum) components. Finally, Dynamic Stiffness parameters are given based on bearing coefficients and *vice versa*. We then show how fluid-induced instability can be readily predicted with these Dynamic Stiffness parameters.

Elliptic Orbit Construction

With x and y probes installed laterally, as shown in Figure 1, any arbitrary elliptical orbit with filtered frequency ω can be constructed in terms of forward and reverse rotating vectors. The rotor response with frequency ω at the probe locations can be given by

$$\boldsymbol{r} = \frac{A}{2} e^{j\alpha}\, e^{j\omega t} + \frac{B}{2} e^{j\beta}\, e^{-j\omega t} \tag{2}$$

where

$$Ae^{j\alpha} = \frac{1}{2}\left(A_x e^{-j\alpha_x} + jA_y e^{-j\alpha_y}\right)$$

$$Be^{j\beta} = \frac{1}{2}\left(A_x e^{j\alpha_x} + jA_y e^{j\alpha_y}\right)$$

A_x, α_x = amplitude (peak-to-peak) and phase lag in Probe x with filtered frequency ω

A_y, α_y = amplitude (peak-to-peak) and phase lag in Probe y with filtered frequency ω

A, α = amplitude and phase lag as shown in Figure 1 with forward frequency ω

B, β = amplitude and phase lag as shown in Figure 1 with reverse frequency $-\omega$

For synchronous $(\omega = \Omega)$ and nonsynchronous $(\omega \neq \Omega)$ response, the filtered orbit will be elliptic whenever a reverse component B exists. As shown in Figure 1, the major axis is half of the forward component A and the reverse component B ; the minor axis is half of A minus B ; and the orbital inclination angle is half of the phase angles α and β.

Fluid Force in an Anisotropic Bearing

While an isotropic bearing leads to a circular orbit response for a symmetric rotor, an anisotropic bearing

MAJOR / MINOR STIFFNESS AND DAMPING AXES DUE TO ANISOTROPIC EFFECT

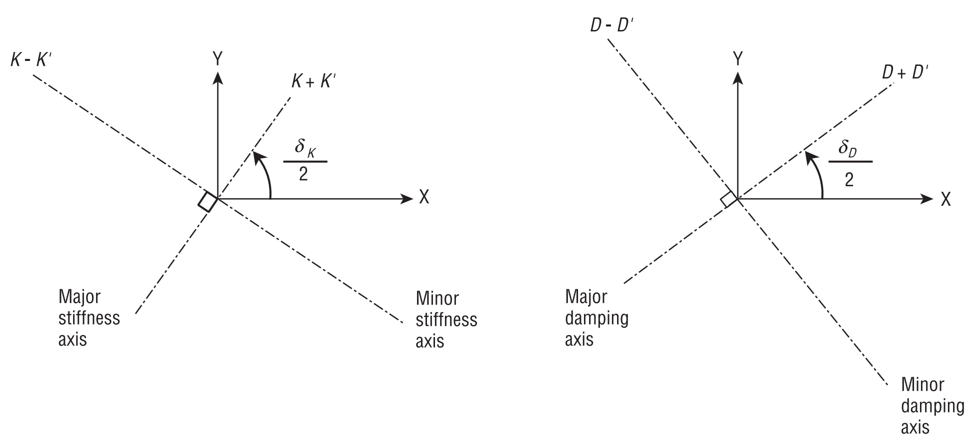

FIGURE 2

results in an elliptical orbit response. As shown previously, the latter causes a combination of both forward and reverse responses. To account for this effect, additional terms are introduced into equation (1). When fluid inertia is neglected, the fluid force in an anisotropic bearing, from equation (1), can be written as

$$-\boldsymbol{F}_f = D\dot{\boldsymbol{r}} + \left(K - j\lambda\Omega D\right)\boldsymbol{r} + \underbrace{D' e^{j\delta_D}\dot{\bar{\boldsymbol{r}}} + K' e^{j\delta_K}\bar{\boldsymbol{r}}}_{\text{anisotropic effect}} \quad \textbf{(3)}$$

where $\bar{r} = x - jy$ denotes the complex conjugate of $\boldsymbol{r}$ (if $\boldsymbol{r}$ rotates at forward frequency ω, for example, $r = r_0 e^{j(\omega t + \alpha_0)}$, then $\bar{r}$ will rotate at reverse frequency -ω, i.e., $r = r_0 e^{-j(\omega t + \alpha_0)}$). Reverse stiffness and damping, K' and D', cause reverse component B as indicated previously. They are oriented at δ_K and δ_D, and vary with the x, y coordinate system orientation (for example, δ_K and δ_D would decrease by 20° if the x, y coordinate system rotates 10° forward). Major/minor stiffness and damping for this anisotropic bearing are related to the anisotropic effect, as shown in Figure 2. The major stiffness axis is oriented $\delta_K/2$ relative to the x-axis with a maximum value $K + K'$, and the minor stiffness axis $\delta_K/2 + 90^\circ$ with a minimum value $K - K'$. The major/minor axes for damping have the same properties as those for stiffness. In general, major/minor axes for the ellipse of a rotor's orbital response, stiffness, and damping, are neither co-linear nor perpendicular. At low speed, however, where damping effects are negligible, the major axis for the ellipse of rotor response is the same as, or co-linear to, the minor axis for stiffness. Note that these major/minor axes exist only if reverse stiffness K' and reverse damping D' exist.

"It is suggested that the Dynamic Stiffness parameters (which give clear insight into instability scenarios), or the bearing coefficients as a function of the Dynamic Stiffness parameters, be used in rotor dynamic analysis."

Dynamic Stiffness parameters for an anisotropic bearing can be determined by performing nonsynchronous perturbation. Relations between the rotor response as a combination of forward and reverse components and the perturbation force can be established in terms of Dynamic Stiffness parameters. Therefore, these Dynamic Stiffness parameters can be obtained through direct and quadrature plots versus perturbation speed.

Transformations between Dynamic Stiffness Parameters and Classical Bearing Coefficients

Dynamic Stiffness parameters (K, D, λ, m_f, K', D', δ_K, δ_D) provide a clear description of bearing characteristics, especially instability margins. Bearing coefficients, however, do not relate the four stiffness components (K_{xx}, K_{xy}, K_{yx}, K_{yy}) to the four damping components (D_{xx}, D_{xy}, D_{yx}, D_{yy}). Thus, it is necessary to establish a transformation between these two expressions.

From equation (3), if the x, y coordinate system is used instead of the complex variable $r = x + jy$, then bearing stiffness and damping matrices can be expressed in terms of Dynamic Stiffness parameters as follows:

$$\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} = \begin{bmatrix} K - \lambda^2\Omega^2 m_f & \lambda\Omega D \\ -\lambda\Omega D & K - \lambda^2\Omega^2 m_f \end{bmatrix} + \underbrace{K'\begin{bmatrix} \cos\delta_K & \sin\delta_K \\ \sin\delta_K & -\cos\delta_K \end{bmatrix}}_{\text{anisotropic effect}} \quad \textbf{(4)}$$

$$\begin{bmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{bmatrix} = \begin{bmatrix} D & 2\lambda\Omega m_f \\ -2\lambda\Omega m_f & D \end{bmatrix} + \underbrace{D'\begin{bmatrix} \cos\delta_D & \sin\delta_D \\ \sin\delta_D & -\cos\delta_D \end{bmatrix}}_{\text{anisotropic effect}} \quad \textbf{(5)}$$

Therefore, one can obtain bearing stiffness and damping coefficients if the Dynamic Stiffness parameters are known [Table 1]. Likewise, one can calculate Dynamic Stiffness parameters if the tabulated

BEARING COEFFICIENTS IN TERMS OF DYNAMIC STIFFNESS PARAMETERS

BEARING COEFFICIENT	EXPRESSION
K_{xx} (N/m, lb/in) =	$K - \lambda^2\Omega^2 m_f + K' \cos\delta_K$
K_{xy} (N/m, lb/in) =	$\lambda\Omega D + K' \sin\delta_K$
K_{yx} (N/m, lb/in) =	$-\lambda\Omega D + K' \sin\delta_K$
K_{yy} (N/m, lb/in) =	$K - \lambda^2\Omega^2 m_f - K' \cos\delta_K$
D_{xx} (N•s/m, lb•s/in) =	$D + D' \cos\delta_D$
D_{xy} (N•s/m, lb•s/in) =	$2\lambda\Omega m_f + D' \sin\delta_D$
D_{yx} (N•s/m, lb•s/in) =	$-2\lambda\Omega m_f + D' \sin\delta_D$
D_{yy} (N•s/m, lb•s/in) =	$D - D' \cos\delta_D$

TABLE 1

DYNAMIC STIFFNESS PARAMETERS IN TERMS OF BEARING COEFFICIENTS

DYNAMIC STIFFNESS PARAMETER	EXPRESSION
λ =	$\dfrac{K_{xy} - K_{yx}}{\Omega(D_{xx} + D_{yy})}$, or $\dfrac{k_{xy} - k_{yx}}{d_{xx} + d_{yy}}$ when nondimensional coefficients k_{ij}, d_{ij} are used
K (N/m, lb/in) =	$\frac{1}{2}(K_{xx} + K_{yy}) + \frac{1}{4}(K_{xy} - K_{yx})\dfrac{D_{xy} - D_{yx}}{D_{xx} + D_{yy}}$
D (N•s/m, lb•s/in) =	$\frac{1}{2}(D_{xx} + D_{yy})$
m_f (kg, lb•s^2/in) =	$\frac{1}{4}\dfrac{(D_{xy} - D_{yx})(D_{xx} + D_{yy})}{K_{xy} - K_{yx}}$
K' (N/m, lb/in) =	$\frac{1}{2}\sqrt{(K_{xx} - K_{yy})^2 + (K_{xy} + K_{yx})^2}$
D' (N•s/m, lb•s/in) =	$\frac{1}{2}\sqrt{(D_{xx} - D_{yy})^2 + (D_{xy} + D_{yx})^2}$
δ_K (deg) =	$\tan^{-1}\left(\dfrac{K_{xy} + K_{yx}}{K_{xx} - K_{yy}}\right)$
δ_D (deg) =	$\tan^{-1}\left(\dfrac{D_{xy} + D_{yx}}{D_{xx} - D_{yy}}\right)$

TABLE 2

bearing coefficients are known [Table 2]. Note that bearing coefficients in handbooks are often given in nondimensional form. For instance, the following nondimensional stiffness and bearing coefficients k_{ij} and d_{ij} are used in a bearing data handbook [3]:

$$k_{ij} = \frac{C_p}{W} K_{ij}, \quad d_{ij} = \frac{C_p \Omega}{W} D_{ij} \qquad \textbf{(6)}$$

where i, j stand for index x or y and W, C_p, and Ω denote bearing load (the radial force), bearing/journal radial clearance, and rotative speed, respectively. Therefore, the fluid average circumferential velocity ratio, λ, can be expressed directly in terms of nondimensional bearing coefficients [Table 2]. As is shown in the table, when cross damping coefficients D_{xy} and D_{yx} are the same, fluid inertia m_f will be zero. This is often the case when using tabulated bearing coefficients, and indicates a negligible amount of fluid inertia. Consequently, fluid stiffness K will be the average value of K_{xx} and K_{yy}. In seals, however, D_{xy} and D_{yx} are no longer the same (often D_{xy} = $-D_{yx}$), so that fluid inertia m_f cannot be neglected. Fluid damping D is always the average value of D_{xx} and D_{yy}. As long as $K_{xx} \neq K_{yy}$ or $K_{xy} \neq -K_{yx}$, reverse stiffness K' will exist. Likewise, if $D_{xx} \neq D_{yy}$ or $D_{xy} \neq -D_{yx}$, reverse damping D' will be nonzero. Their orientation can be calculated and is related to the major/minor stiffness or damping axis.

Summary

This article has established several compelling reasons for the use of Dynamic Stiffness parameters (K, D, λ, m_f, K', D', δ_K, δ_D) instead of less meaningful classical bearing coefficients (K_{xx}, K_{xy}, K_{yx}, K_{yy}, D_{xx}, D_{xy}, D_{yx}, D_{yy}) to model an anisotropic bearing. It has been demonstrated that rotor elliptical synchronous or nonsynchronous response, due to anisotropic bearings, is a combination of forward and reverse components, which can be measured with two probes. Parameters K', D', δ_K, δ_D that relate to major/minor stiffness and damping axes, cause the reverse component. The relationship between the Dynamic Stiffness parameters and the bearing coefficients has been established. One can easily obtain the bearing coefficients in terms of the Dynamic Stiffness parameters, or the Dynamic Stiffness parameters in terms of the bearing coefficients. It is suggested that the Dynamic Stiffness parameters (which give clear insight into instability scenarios), or the bearing coefficients as a function of the Dynamic Stiffness parameters, be used in rotor dynamic analysis. The key parameter, λ, which contributes to the destabilizing cross stiffness term, K_{xy} or K_{yx} , can be directly calculated from nondimensional bearing coefficients. ORBIT

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